THE PRESENTATION OF THE NONSTEADY TEMPERATURE FIELD OF A STEAM TURBINE ROTOR IN GENERALIZED FUNCTIONS

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Results of investigation into the possibility of expressing the temperature field of a turbine rotor in terms of generalized functions of temperature for use in the analysis of nonsteady thermal processes are presented.

Investigation of the thermal state of steam turbine rotors by approximate analytical methods and by electrical simulation has been the subject of numerous publications. The most comprehensive survey of this problem is given in [1].

It must be kept in mind that the assessment of the thermal state of rotors during starts from various states necessitates the investigation of a virtually infinite number of transitional conditions. Investigation of the thermal state of even a single type of rotor requires a lengthy programming of an analog computer.

There exists, therefore, an undoubted interest in devising a method of solving the problem on an analog computer which would subsequently permit the determination of temperature variation at the most characteristic points under any transitional conditions and during starts from any initial states without resorting to further simulation.

The theory of thermal conductivity [2] shows that in problems of simple heating (cooling) of bodies of conventional form, it is possible to find for individual points of such bodies certain functions of temperature $\overline{\theta} = f(\mathrm{Bi}, \mathrm{Fo})$ which we shall call generalized functions.

Although in one-dimensional problems related to bodies of conventional form with constant boundary conditions $\overline{\theta} = f(\mathrm{Bi}, \mathrm{Fo})$, it may not be so in the case of a rotor. In problems of heating (cooling) of rotors, owing to the expansion process, boundary conditions vary from stage to stage, and it is not possible to assert a priori that functions $\overline{\theta}$ of only Bi and Fo, derived from the heat exchange conditions at the surface in the vicinity of a given point, can be found when boundary conditions at individual characteristic points of a rotor are constant.

However, the fact that the medium temperature and the coefficients of heat exchange at the surface of a rotor change smoothly from stage to stage supports the assumption of the possibility of determining the form of function $\overline{\theta}$ for characteristic points of a rotor from results of simulation on an analog computer of a number of heating modes with fixed boundary conditions corresponding to certain part-load operations and for different initial parameters of steam at the turbine inlet.

The working part of a multistage rotor may be considered as consisting of consecutively connected similar elements (see shaded section in Fig. 1).

To find the form of function $\overline{\theta}$ for each characteristic point of the object under investigation it is necessary to determine the relative importance of the effect of all factors on the temperature at a given point. For a rotor subjected to heat exchange along its external surface only, it is natural to expect the heat exchange at the surface in proximity to the considered point to be of paramount importance. For like points of similar elements, e.g., points 100, 98, 96, ..., 88, and 86 in the rotor bore, or points 49, 53, 57, ..., 73, and 77 along the external generatrix of diaphragm seal sections, etc., (see Fig. 1), we determine a

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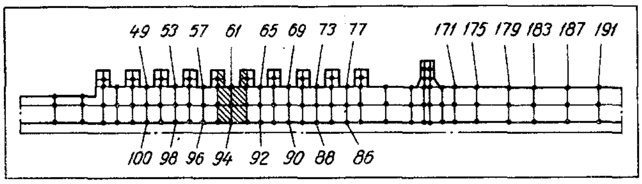

Fig. 1. Diagram of a multistage rotor of a steam turbine.

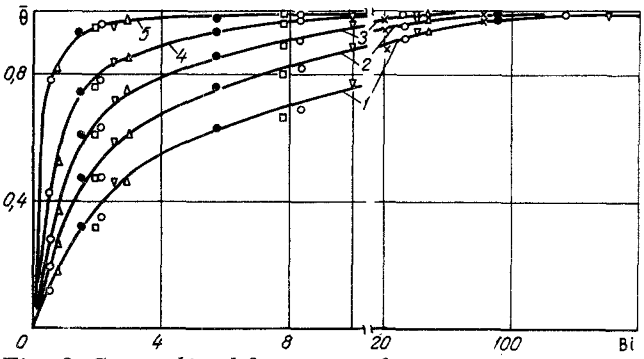

Fig. 2. Generalized functions of temperature $\theta = f(\mathrm{Bi})$ with Fo as parameter for points of the rotor surface in the region of diaphragm seals: 1) Fo = 0.04; 2) Fo = 0.08; 3) Fo = 0.16; 4) Fo = 0.32; 5) Fo = 1.2; ○) point 65; ∇) point 73; Δ) point 77; □) point 61; ×) point 49; ●) point 53; τ) h.

number of numerical values of functions $\bar{\theta}$ at specific instants of time from results of analog computation as

$$\bar{\theta} = \frac{U - U_r}{U_m - U_r}.$$

The difference between the medium maximum temperature at the boundary of the investigated zone and the minimum temperature of the medium or body possible in the simulation of this problem was taken as 100% of the potential.

It was found that for various Bi and fixed Fo $\bar{\theta}$ at like points of the same rotor elements fit well a single curve. As an example, curves of $\bar{\theta} = f(\mathrm{Bi})$ are shown in Fig. 2 for several values of Fo and points 49, 53, 57, 61, 65, 73, and 77 of the outer generatrix of diaphragm seals. We point out that different conditions of heat exchange (the Biot number and the temperature of the medium) at the considered points were obtained in different heating problems, and are also due to the natural variation of heat exchange conditions along the working part (of the rotor). Data pertaining to Figs. 2 and 3 are summarized in Table 1.

Similar generalized functions $\bar{\theta} = f(\mathrm{Bi})$ with Fo as parameter apply, also, to other similar points along rotor stages and at end seals.

Using the method of least squares and knowing a number of values of function $\bar{\theta}$ for certain values of Fo and given Bi, it is obviously possible to determine the form of the approximating function $\bar{\theta} = f(\mathrm{Fo})$ with Bi as parameter (see, e.g., Fig. 3). We note that when the form of function $\bar{\theta} = f(\mathrm{Fo})$ with Bi as a fixed parameter is determined in this way, an interpolation of this function with respect to its values at other Bi becomes necessary.

If the initial temperature distribution is either uniform or an analytic function, which corresponds to a stationary thermal state and constant temperature of the medium, the variation of the generalized function of temperature at a given point is independent of the medium temperature and of the initial temperature distribution. The analysis of results of this investigation confirms the validity of this statement also for bodies of complicated shapes. Let us consider individual points of curves in Fig. 2, where, e.g., experimentally obtained temperatures $t_m = 551$°K of the medium and $t_r = 473$°K for Bi = 1.95, $t_m = 563$°K and $t_r = 408$°K for Bi = 5.69, $t_m = 622$°K and $t_r = 599$°K for Bi = 8.3, etc., correspond to points along curve 3.

Since the investigations have shown that at like points of similar elements of a multistage rotor the generalized functions $\bar{\theta} = f(\mathrm{Bi, Fo})$ are identical, the investigation of the thermal state of a rotor requires the construction of a comparatively limited number of sets of generalized functions. Moreover, since at points of plane cross sections normal to the axis $\bar{\theta} = f(\mathrm{Bi, Fo})$, the temperature field of the rotor can be determined by calculating the temperature variation at n plane sections.

A comparison of generalized functions of temperature for the front-end seal region with those for an infinitely long cylinder [2] is of interest. It will be seen that the generalized functions of temperature for an infinitely long cylinder and those for the end-seal region (see Fig. 3A) are virtually identical, since the thermal state of the extended part of the latter (points 175-191 in Fig. 1) approaches that of an infinitely long cylinder. Temperature differences at selected points of a turbine rotor can be readily estimated for any conditions of simple heating by comparing the variation of $\bar{\theta} = f(\mathrm{Bi, Fo})$ at such points.

The generalized functions of temperature for characteristic points of the investigated region, derived by simulating a number of problems of simple heating and cooling, make at the same time possible the calculation of temperature variation at such points for any conditions of complex heating.

TABLE 1. Summary of Data on Boundary and Initial Conditions at Characteristic Points of a Rotor for which Generalized Functions of Temperature are Shown in Figs. 2 and 3

	Point 65		Point 73		Point 75		Point 61		Point 49		Point 53	
	Bi	t_m, °K	Bi	t_m, °K	Bi	t_m, °K	Bi	t_m, °K	Bi	t_m, °K	Bi	t_m, °K
To Fig. 2	0,5	453	2,5	503	0,75	453	1,95	551	20,5	677	1,42	443
	2,09	565	10	606	2,94	607	7,8	605	82	563	5,69	563
	8,36	622	40	693	47,2	667	—	—	—	—	90,9	673
	33,5	633	160	743	—	—	—	—	—	—	—	—
	134	703	—	—	—	—	—	—	—	—	—	—

	Point 183			Point 175			Point 187			Point 191		
	Bi	t_r, °K	t_m, °K	Bi	t_r, °K	t_m, °K	Bi	t_r, °K	t_m, °K	Bi	t_r, °K	t_m, °K
To Fig. 3a	130	576	718	9,9	473	544	28	545	688	0,61	413	663
	8,1	473	603	2,5	569	596	—	—	—	—	—	—

	Point 100			Point 98			Point 86			Point 88		
	Bi	t_r, °K	t_m, °K	Bi	t_r, °K	t_m, °K	Bi	t_r, °K	t_m, °K	Bi	t_r, °K	t_m, °K
To Fig. 3b	20,5	413	681	90	521	703	12	548	734	10	473	719
	5,15	333	512	5,6	333	527	2,94	413	507	2,5	413	498
	1,3	413	435	1,4	473	655	—	—	—	—	—	—

The method of solving complex heating problems with time-variable boundary conditions of the third kind by using generalized functions is based on the following proposition. Investigations carried out on models of various steam turbine structural elements have shown an absence of distortion of the temperature field, when a piece-wise function, with the approximating α_{3i} not exceeding ±15% of actual values, is substituted for the continuously varying function $\alpha = f(\tau)$. Hence it is possible to consider the problem of complex heating during the time interval of constant α_{31} as one of simple heating with step-wise transition to new conditions at the boundaries in the next following time interval.

Noting that, as shown by investigations, the relative temperature $\bar{\theta}$ is independent of the initial and the medium temperatures, from the value of $\bar{\theta}_1 = f(Bi_1, Fo_1)$ obtaining at the end of the first time interval τ_1 we determine a fictitious time τ_{2f} corresponding to the same $\bar{\theta}_1$ but at a new value of $\alpha_{32}(Bi_2)$. Then from function $\bar{\theta} = f(Bi_2, Fo)$ we determine the generalized temperature at the given point and in the interval of time $\Delta\tau_2$ from τ_{2f} to $\tau_2 = \tau_{2f} + \Delta\tau_2$, and so on. The duration of the time interval $\Delta\tau_2$ corresponds to the sector of constant α_{32}.

The variation of temperature at certain points of a rotor under starting conditions with sliding parameters computed by generalized functions and by electrical analog methods, as well as the variation of the medium temperature and of the heat exchange coefficient in the neighborhood of certain points of the rotor are shown in Fig. 4. The comparison shows that at various points of the rotor surface and inside it the discrepancy between computed values and those measured in direct electrical simulation of this problem do not exceed the limits of accuracy of solutions on grid models.

This leads to the conclusion that generalized functions of temperature make possible the calculation of temperature variation at characteristic points of a region for any law governing boundary conditions at the surfaces of heat exchange.

Generalized functions of temperature permit, moreover, the calculation of the thermal elongation of a rotor under any transitory conditions. However, the calculation of thermal elongation necessitates the construction of mean integral generalized functions for (individual) cross sections. This is done in the same manner as the calculation of actual temperature, except that mean integral generalized functions are used. In this case, as in the calculation of the temperature field, a piece-wise linear function $\alpha = f(\tau)$ with deviations of α_{3i} not exceeding ±15% of actual values is constructed for a number of instants of time from obtained values of heat exchange coefficients at characteristic points of the rotor surface. Having determined $\alpha_{3i}(Bi_i)$ for the piece-wise linear function $\alpha = f(\tau)$ and the duration of constancy intervals, we plot the curve of $\bar{\theta}_g = f(\tau)$ calculated for a cross section from the mean integral functions $\bar{\theta} = f(Bi, Fo)$. Knowing the law of variation of the relative temperature $\bar{\theta}_g$ in a section and the temperature variation of the medium $t_m = f(\tau)$, we determine the mean integral temperature of a cross section at any instant of time from

$$t(\tau) = \bar{\theta}_g(\tau)[t_m(\tau) - t_r] + t_r$$

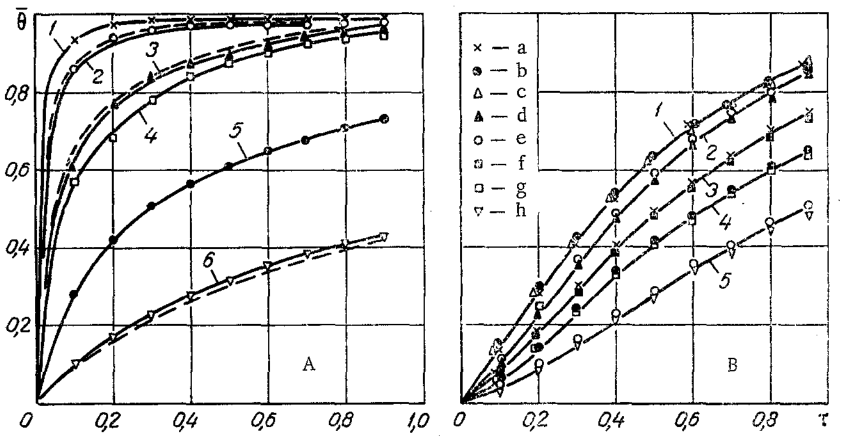

Fig. 3. A) Generalized functions of temperature $\bar{\theta}$ = f(Fo) for points at the rotor surface in the region of front-end seal with Bi as parameter [1) Bi = 130; 2) Bi = 28; 3) Bi = 9.9; 4) Bi = 8.1; 5) Bi = 2.5; 6) Bi = 0.61]. Dashed lines denote curves calculated in [2]. B) The same functions for points of the rotor bore in the region of diaphragm seals with Bi as parameter [1) *a* is point 100; b is point 98; c is point 92 (Bi > 20); 2) d is point 88; e is point 86 (Bi = 10-12); 3) f is point 100; *a* is point 98 (Bi ≈ 5.5); 4) b is point 86; g is point 88 (Bi ≈ 2.7); 5) h is point 100; e is point 98 (Bi ≈ 1.4)].

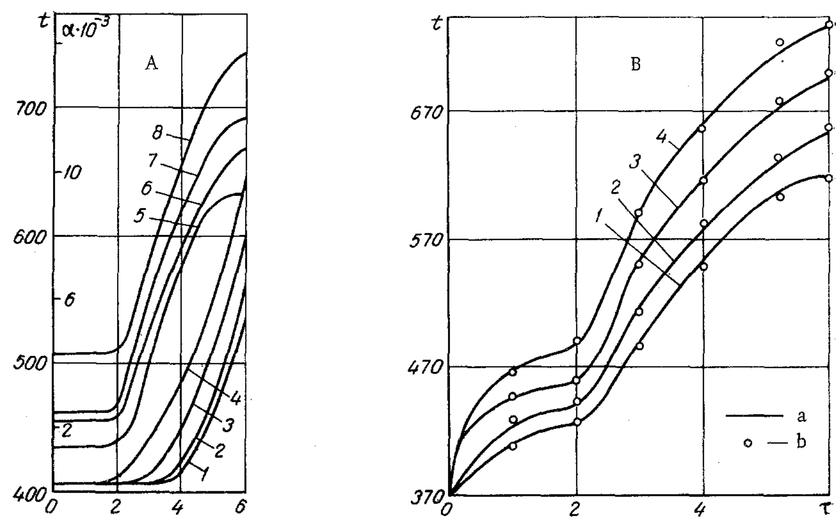

Fig. 4. Temperature variation at characteristic points of a rotor under starting conditions with sliding parameters. A) Variation of the medium temperature and of the intensity of heat exchange in the vicinity of certain points of the rotor surface [(1)-(4) and (5)-(8) are, respectively, the heat exchange intensities and the temperatures of the medium at points: 49, 57, 65, and 171]. B) Variation of temperature at certain points of the rotor calculated by generalized formulas and electrical simulation under considered conditions [1) point 100; 2) point 96; 3) point 65; 4) point 171]; calculated curves are denoted by *a*; b denotes temperatures measured for simulation, t in °K, α in $W/m^2 \cdot deg$.

The calculation of thermal elongation from known mean integral temperatures presents no further difficulties.

Owing to technical complexity of temperature measurements, these are limited in practical investigations of steam turbines to the determination of temperature distribution along the central bore of the rotor.

Obviously, such temperature measurements do not provide exhaustive data on the thermal state of a rotor under transient conditions. The presentation of temperature fields in generalized functions widens the possibility of using the results of temperature measurements in central bores of steam turbine rotors for solving inverse problems of unstable heat conduction, and the subsequent determination of the temperature field corresponding to actual conditions of heat exchange at the surface during a transient mode of operation. It should be noted, however, that this requires, in addition to measuring the temperature in the central bore of the rotor, the temperature of steam at the same cross sections of the working part of the rotor to be measured, since the determination of the relative temperature $\overline{\theta}_g$ necessitates the knowledge of the temperature variation at a given point, as well as that of the temperature of the medium in the vicinity of that point.

The curves of generalized functions of temperature appearing in Fig. 3B for points of the rotor center bore in the zone of diaphragm seals show that an error of 1% in the definition of the relative temperature $\overline{\theta}_g$, even for an intensity of heat exchange corresponding to $\alpha/\lambda \approx 5.0\ m^{-1}$, results in an error of an order of 10% in the determination of α. For a heat exchange intensity increased to $\alpha/\lambda \approx 30\ m^{-1}$ the same error in the magnitude of the relative temperature leads to an error of the order of 20% in the definition of α. However, measurements of temperature inside the bore as the heat exchange intensity is increased up to $\alpha/\lambda \geq 50\ m^{-1}$ lead to the conclusion that the heat intensity at the surface is above this level, since further increase of the heat exchange intensity has virtually no effect on temperature variation in the rotor bore.

NOTATION

U	is the instantaneous potential at a given point of the model, %;
U_r	is the initial potential at a given point of the model corresponding to a uniform distribution or a steady state, %;
U_m	is the potential of the medium in the vicinity of a given point of the model, %;
t_r	is the initial temperature at a given point corresponding to a uniform distribution or a steady state;
t_m	is the temperature of the medium in the vicinity of the given point;
$Bi = \alpha r/\lambda$	is the Biot number, where r is the external radius of the rotor taken as the reference dimension;
α	is the heat exchange coefficient;
α_{3i}	is the approximating value of the heat exchange coefficient whose deviation from actual values does not exceed ±15%.

LITERATURE CITED

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